(155)

$$F_g = \mathcal{G}\frac{M_1 M_2}{r^2} = 6{,}67\cdot10^{-11}\frac{8\cdot8}{0{,}5^2} \doteq 1{,}28\cdot10^{-8}\ N$$

(156)

$$F_g = \mathcal{G}\frac{M_1 M_2}{r^2} = 6{,}67\cdot10^{-11}\frac{2000\cdot2000}{0{,}8^2}$$

$$F_g = 4{,}17\cdot10^{-4}\ N$$

(157)

$$F_g = \mathcal{G}\frac{M_1 M_2}{r^2} \Rightarrow M_1 = \frac{F_g\cdot r^2}{\mathcal{G} M_2}$$

$$M_1 = \frac{4{,}2\cdot10^{23}\cdot(7{,}8\cdot10^{11})^2}{6{,}67\cdot10^{11}\cdot 2\cdot10^{30}} \doteq 1{,}9\cdot10^{27}\ kg$$

$$a_{JUPITER} = \frac{F_g}{m_J} = \frac{4{,}2\cdot10^{23}}{1{,}9\cdot10^{27}} = 2{,}2\cdot10^{-4}\ m\cdot s^{-2}$$

$$a_{SŁOŃCE} = \frac{4{,}2\cdot10^{23}}{2\cdot10^{30}} = 2{,}1\cdot10^{-7}\ m\cdot s^{-2}$$

(158)

$g_1 = 9{,}8\ m\cdot s^{-2}$

$g_2 = 8\ m\cdot s^{-2}$

$r = R_z + h$

$$g_2 = \mathcal{G}\frac{M_z}{r^2} \Rightarrow r^2 = \mathcal{G}\frac{M_z}{g_2}$$

$$R_z^2 = \mathcal{G}\frac{M_z}{g_1} \Rightarrow R_z^2\cdot g_1 = \mathcal{G} M_z$$

$$r^2 = \frac{R_z^2\cdot g_1}{g_2}$$

$$h = r - R_z = \sqrt{\frac{R_z^2\cdot g_1}{g_2}} - R_z$$

$$h = \sqrt{\frac{6370000^2\cdot 9{,}8}{8}} - 6370000 \doteq 680\ km$$

159

$$g = \varkappa \frac{M}{r^2} \qquad g_z = \varkappa \frac{M_z}{R_z^2}$$

$$g_v = \varkappa \frac{0{,}815 M_z}{(0{,}97 \cdot R_z)^2}$$

$$\frac{g_z}{g_v} = \frac{\varkappa M_z \cdot (0{,}97 R_z)^2}{R_z^2 \cdot \varkappa \cdot 0{,}815 M_z}$$

$$\frac{g_z}{g_v} = \frac{0{,}97^2}{0{,}815} \doteq 1{,}15 \Rightarrow g_v = \frac{g_z}{1{,}15}$$

$$g_v = \frac{9{,}8}{1{,}15} \doteq 8{,}52 \ m\,s^{-2}$$

160

$$g_1 = \varkappa \frac{M_z}{R_z^2}$$

$$g_2 = \varkappa \frac{M_z}{(1{,}5 \cdot R_z)^2}$$

$$\frac{g_1}{g_2} = \frac{\varkappa M_z (1{,}5 \cdot R_z)^2}{\varkappa M_z R_z^2}$$

$$\frac{g_1}{g_2} = 1{,}5^2 = 2{,}25$$

$$g_2 = \frac{g_1}{2{,}25}$$

161

$$v = v_0 - gt \Rightarrow h = v_0 \cdot t - \frac{1}{2} g t^2$$

$$h = 30 \cdot 2 - \frac{1}{2} \cdot 10 \cdot 2^2 = 60 - 20 = \underline{40\,m}$$

$$v = 30 - 10 \cdot 2 = \underline{10\,m\,s^{-1}}$$

MAXIMÁLNÍ VÝŠKA $v = 0 \Rightarrow v_0 = gt \Rightarrow t = \frac{v_0}{g}$

$$h_{max} = v_0 \cdot \frac{v_0}{g} - \frac{1}{2} g \left(\frac{v_0}{g}\right)^2 = 30 \cdot \frac{30}{10} - \frac{1}{2} \cdot 10 \left(\frac{30}{10}\right)^2$$

$$h_{max} = 90 - 45 = \underline{45\,m}$$

(162) $v = v_0 - gt \qquad h = v_0 \cdot t - \frac{1}{2} g t^2$

PRO h_{max} PLATÍ $v = 0 \Rightarrow t = \frac{v_0}{g}$

$$h_{max} = v_0 \cdot \frac{v_0}{g} - \frac{1}{2} g \frac{v_0^2}{g^2} = \frac{v_0^2}{g} - \frac{1}{2} \frac{v_0^2}{g}$$

$$h_{max} = \frac{1}{2} \frac{v_0^2}{g} \Rightarrow v_0 = \sqrt{2 g h_{max}}$$

$$v_0 = \sqrt{2 \cdot 10 \cdot 20} = \sqrt{400} = 20 \text{ m.s}^{-1}$$

NA MĚSÍCI

$$t = \frac{v_0}{\frac{1}{6} g} = \frac{6 v_0}{g} = \frac{6 \cdot 20}{10} = 12 \text{ s}$$

$$h_{max} = 20 \cdot 12 - \frac{1}{2} \cdot \frac{1}{6} g \cdot 12^2 = 240 - \frac{1}{12} g \cdot 12^2$$

$$h_{max} = 240 - g \cdot 12 = 240 - 120 = 120 \text{ m}$$

(163)

$$h = \frac{1}{2} g t^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$$

$$v = g \cdot t = g \cdot \sqrt{\frac{2h}{g}} = 9{,}8 \sqrt{\frac{2 \cdot 4}{9{,}8}}$$

$$v = 8{,}85 \text{ m.s}^{-1}$$

$$h = v_0 \cdot t - \frac{1}{2} g t^2$$

PRO h_{max} PLATÍ $v = 0 \quad t = \frac{v_0}{g}$

$$h_{max} = \frac{v_0^2}{g} - \frac{1}{2} \frac{v_0^2}{g} \Rightarrow v_0 = \sqrt{2 g h_{max}} = \sqrt{2 \cdot 9{,}8 \cdot 2} = 6{,}26 \text{ m.s}^{-1}$$

164 PODLE PŘÍKLADU 163

$$v = g\sqrt{\frac{2h}{g}} = \sqrt{2hg}$$ — RYCHLOST DOPADU

RYCHLOST ODRAZU JE $\frac{1}{2}v = \frac{\sqrt{2hg}}{2} = v_0$

PRO t PLATÍ (MAXIMÁLNÍ VÝŠKA) $t = \sqrt{\frac{v_0}{g}} = \sqrt{\frac{\sqrt{2hg}}{2g}}$

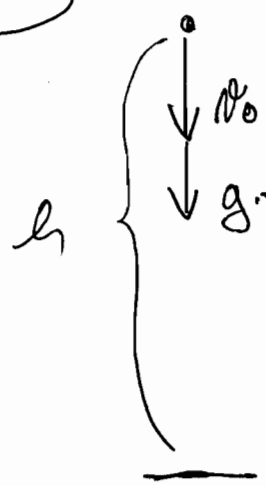
$$h_{max} = v_0 \cdot t - \frac{1}{2} g t^2 = \frac{\sqrt{2hg}}{2} \cdot \sqrt{\frac{\sqrt{2hg}}{2g}} -$$

$$- \frac{1}{2} g \sqrt{\frac{2hg}{2g}}$$

$$h_{max} = \frac{\sqrt{2 \cdot 20 \cdot 9{,}8}}{2} \cdot \sqrt{\frac{\sqrt{2 \cdot 20 \cdot 9{,}8}}{2 \cdot 9{,}8}} - \frac{1}{2} \cdot 9{,}8 \frac{\sqrt{2 \cdot 20 \cdot 9{,}8}}{2 \cdot 9{,}8}$$

$$h_{max} = 9{,}95 - 4{,}95 = \underline{5\ m}$$

165



$$v = v_0 + gt$$

$$h = v_0 t + \frac{1}{2} g t^2 \Rightarrow t$$ — JE TO KVADRATICKÁ ROVNICE

$$\left(\frac{1}{2} g t^2 + v_0 t - h = 0\right)$$

$$t_{1,2} = \frac{-v_0 \pm \sqrt{v_0^2 - 4 \cdot \frac{1}{2} g \cdot (-h)}}{2 \cdot \frac{1}{2} g}$$

$t_1 = 3\ s$ BEREME

$t_2 = -6\ s$

RYCHLOST DOPADU $v = 15 + 10 \cdot 3 = \underline{45\ m \cdot s^{-1}}$

(167)

$$h = v_0 \cdot t - \frac{1}{2} g t^2$$

$v = v_0 - gt \Rightarrow$ PRO h_{max} JE $t = \frac{v_0}{g}$

$$h = v_0 \cdot \frac{v_0}{g} - \frac{1}{2} g \left(\frac{v_0}{g}\right)^2$$

$$h = \frac{v_0^2}{g} - \frac{1}{2} \frac{v_0^2}{g} = \frac{1}{2} \frac{v_0^2}{g}$$

BOD 3 JE MÍSTEM SRÁŽKY

PLATÍ: $s_1 + s_2 = h$ A t JE STEJNÉ PRO TĚLESO 1 A 2

$$\frac{1}{2} g t^2 + v_0 \cdot t - \frac{1}{2} g t^2 = \frac{1}{2} \frac{v_0^2}{g}$$

$$v_0 \cdot t = \frac{1}{2} \frac{v_0^2}{g} \Rightarrow t = \frac{1}{2} \frac{v_0}{g}$$

$$t = \frac{1}{2} \frac{9{,}8}{9{,}8} = \underline{0{,}5\ s}$$

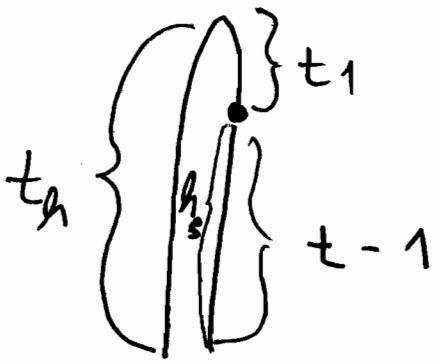
$$h_{max} = \frac{1}{2} \cdot \frac{9{,}8^2}{9{,}8} = \underline{4{,}9\ m} \Rightarrow$$ VÝŠKA SRÁŽKY

JE $h_1 = v_0 t - \frac{1}{2} g t^2 = 9{,}8 \cdot 0{,}5 - \frac{1}{2} 9{,}8 \cdot 0{,}5^2$

$$h_1 = s_2 = \underline{3{,}675\ m}$$

(168)

t – CELKOVÝ ČAS LETU 1 TĚLESA KE SRÁŽCE



t_h – ČAS VYSTOUPÁNÍ 1 TĚLESA DO MAXIMÁLNÍ VÝŠKY

$$t = t_1 + t_h$$

$$t - 1 + t_1 = t_h$$

$$t_1 + t_h - 1 + t_1 = t_h$$

$$2t_1 = 1$$

$$t_1 = \frac{1}{2}$$

$$\underline{t_1 = 0{,}5\,s}$$

$$t_h = \frac{v_0}{g} \quad \left(\text{V MAXIMÁLNÍ VÝŠCE JE } v = 0 \Rightarrow v = v_0 - gt = 0 \Rightarrow t = \frac{v_0}{g} \right)$$

$$t_h = \frac{24{,}5}{9{,}8} = \underline{2{,}5\,s}$$

CELKOVÝ ČAS JE $t = t_1 + t_h = 0{,}5 + 2{,}5$

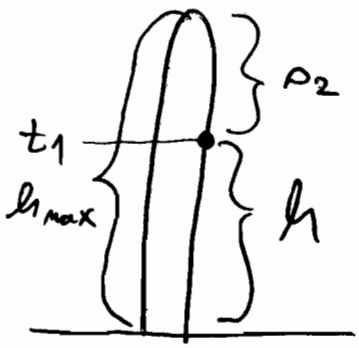
$$\underline{t = 3\,s}$$

ČAS SRÁŽKY VZHLEDEM K DRUHÉMU TĚLESU JE $t - 1 = 3 - 1 = \underline{2\,s}$

VÝŠKA SRÁŽKY $h_s = v_0 t_s - \frac{1}{2} g t_s^2 = 24{,}5 \cdot 2 - \frac{1}{2} \cdot 9{,}8 \cdot 2^2$

$$\underline{h_s = 29{,}4\,m}$$

(169)



PRO ČAS t_1

$$h_1 = v_0 . t_1 - \frac{1}{2} g t_1^2$$

$$h_1 = h_{max} - s_2$$

PRO h_{max} JE $t_m = \frac{v_0}{g}$

PRO s_2 PLATÍ

$$s_2 = \frac{1}{2} g t_3^2$$

$$h_{max} = \frac{1}{2} \frac{v_0^2}{g}$$

$$t_3 = t_2 - t_m$$

$$v_0 t_1 - \frac{1}{2} g t_1^2 = \frac{1}{2} \frac{v_0^2}{g} - \frac{1}{2} g \left(t_2 - \frac{v_0}{g}\right)^2$$

$$v_0 t_1 - \frac{1}{2} g t_1^2 = \frac{1}{2} \frac{v_0^2}{g} - \frac{1}{2} g \left(t_2^2 - 2 t_2 \frac{v_0}{g} + \left(\frac{v_0}{g}\right)^2\right)$$

$$v_0 t_1 - \frac{1}{2} g t_1^2 = \cancel{\frac{1}{2} \frac{v_0^2}{g}} - \frac{1}{2} g t_2^2 + g t_2 v_0 - \cancel{\frac{1}{2} \frac{v_0^2}{g}}$$

$$v_0 t_1 - \frac{1}{2} g t_1^2 = -\frac{1}{2} g t_2^2 + t_2 v_0$$

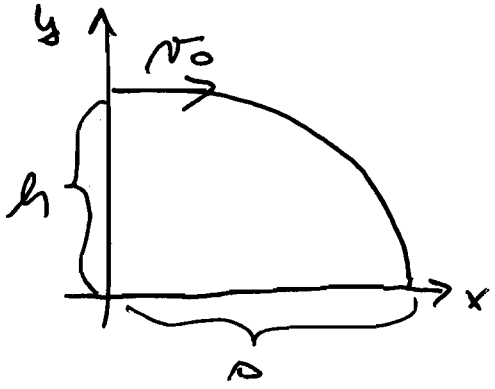
$$\frac{1}{2} g (t_2^2 - t_1^2) = v_0 (t_2 - t_1)$$

$$v_0 = \frac{g (t_2^2 - t_1^2)}{2 (t_2 - t_1)}$$

$$v_0 = \frac{9{,}8 . (2{,}4^2 - 1{,}8^2)}{2 (2{,}4 - 1{,}8)} = \underline{20{,}58 \ m s^{-1}}$$

$$h_1 = 20{,}58 . 1{,}8 - \frac{1}{2} 9{,}8 . 1{,}8^2 = \underline{21{,}17 \ m}$$

170



ČAS DOPADU

$$h = \frac{1}{2} g t^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$$

$$t = \sqrt{\frac{2 \cdot 19{,}2}{9{,}8}} = \underline{2\,s}$$

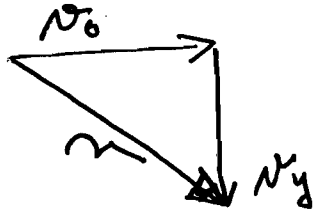
VZDÁLENOST DOPADU $s = v_0 \cdot t$

$$s = 15 \cdot 2 = \underline{30\,m}$$

DOPADOVÁ RYCHLOST VE SMĚRU y JE $v_y = g \cdot t$

$$v = 9{,}8 \cdot 2 = \underline{19{,}6\,m \cdot s^{-1}}$$

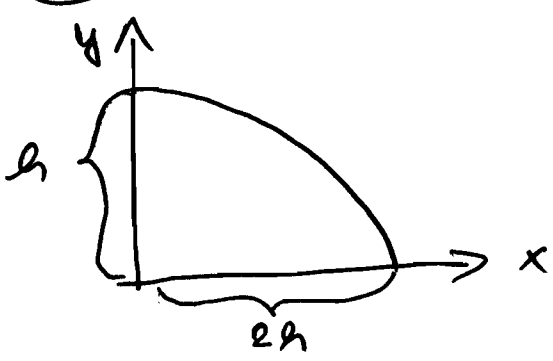
CELKOVÁ RYCHLOST v JE



$$v = \sqrt{v_0^2 + v_y^2}$$

$$v = \sqrt{15^2 + 19{,}6^2} \doteq \underline{24{,}7\,m\,s^{-1}}$$

171



$$h = \frac{1}{2} g t^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$$

$$s = v_0 \cdot t = 2h$$

$$v_0 \cdot \sqrt{\frac{2h}{g}} = 2h$$

$$v_0 = 2h \cdot \sqrt{\frac{g}{2h}}$$

OBECNĚ

$$v_0 = n\,h \sqrt{\frac{g}{2h}} = n \sqrt{\frac{h^2 g}{2h}}$$

$$\underline{v_0 = n \sqrt{\frac{hg}{2}}}$$

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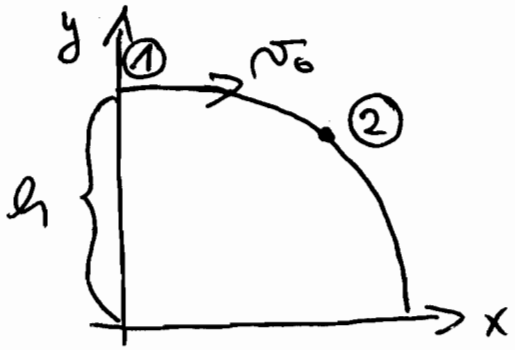


PRO ① PLATÍ

$$E_k = \frac{1}{2} m v_0^2$$ *

PRO ② PLATÍ (ČAS 2 s)

RYCHLOST VE SMĚRU x JE v_0

RYCHLOST VE SMĚRU y JE $v_y = g \cdot t = 10 \cdot 2 = \underline{20\ m\,s^{-1}}$

KINETICKÁ ENERGIE JE $E_k = \frac{1}{2} m v^2$

$$v = \sqrt{v_0^2 + v_y^2} \qquad E_k = \frac{1}{2} m (v_0^2 + v_y^2)$$

ZA ČAS 2 s SE TĚLESO DOSTANE DO VÝŠKY $h_2 = h - \frac{1}{2} g t^2 = 80 - \frac{1}{2} \cdot 10 \cdot 2^2 = \underline{60\ m}$

POTENCIÁLNÍ ENERGIE E_p JE $E_p = m \cdot g \cdot h_2$

CELKOVÁ ENERGIE PRO ②

$$E = E_k + E_p = \frac{1}{2} m (v_0^2 + v_y^2) + m \cdot g \cdot h_2$$

$$E = \frac{1}{2} 0{,}5 (20^2 + 20^2) + 0{,}5 \cdot 10 \cdot 60 = 200 + 300$$

$$E = \underline{500\ J}$$

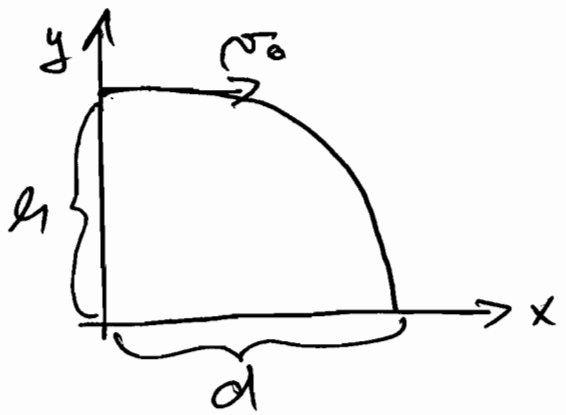
CELKOVÁ ENERGIE PRO ①

$$E = E_k + E_p$$

$$E = \frac{1}{2} m v_0^2 + mgh = \frac{1}{2} \cdot 0{,}5 \cdot 20^2 + 0{,}5 \cdot 10 \cdot 80$$

$$E = 100 + 400 = \underline{500\ J}$$

(173)



$$h = \frac{1}{2} g t^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$$

$$d = v_0 \cdot t = v_0 \cdot \sqrt{\frac{2h}{g}}$$

$$d = v_0 \sqrt{\frac{2h}{g}}$$

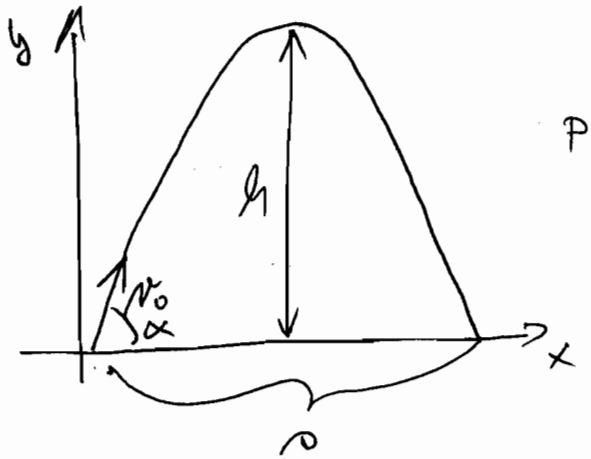
$$v_0 = \frac{d}{\sqrt{\frac{2h}{g}}}$$

a) $$d_1 = \frac{d}{\sqrt{\frac{2h}{g}}} \cdot 4 \cdot \sqrt{\frac{2h}{g}} = \underline{4d}$$

b) $$4h = \frac{1}{2} g t^2 \Rightarrow t = \sqrt{\frac{8h}{g}} = 2\sqrt{\frac{2h}{g}}$$

$$d_2 = \frac{d}{\sqrt{\frac{2h}{g}}} \cdot 2 \cdot \sqrt{\frac{2h}{g}} = \underline{2d}$$

(177)



$v_y = v_0 \sin\alpha$

$v_x = v_0 \cos\alpha$

PRO h PLAT↓

$$h = v_y t - \frac{1}{2} g t^2$$

PRO s PLAT→

$$s = v_x t$$

$$h = s$$

$$v_y t - \frac{1}{2} g t^2 = v_x \cdot t$$

$$v_0 \sin\alpha \cdot t - \frac{1}{2} g t^2 = v_0 \cos\alpha \cdot t$$

$$\frac{1}{2} g t^2 = \frac{1}{2} v_y \cdot t$$

$$v_0 \sin\alpha \cdot t - \frac{1}{2} v_0 \sin\alpha \cdot t = v_0 \cdot \cos\alpha \, t$$

$$\frac{1}{2} v_0 \sin\alpha \cdot t = v_0 \cos\alpha \, t$$

$$\frac{\sin\alpha}{\cos\alpha} = 2 = \operatorname{tg}\alpha \Rightarrow \alpha \doteq \underline{63{,}43^\circ}$$

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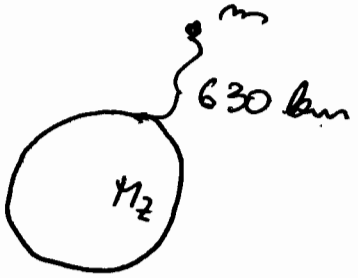
PLATÍ!



$F_o = F_g$

$$\gamma \frac{M_z \cdot m}{r^2} = m r \omega^2 \qquad \omega = \frac{v}{r}$$

$$\gamma \frac{M_z m}{r^2} = m \frac{v^2}{r}$$

$$\Rightarrow v = \sqrt{\gamma \frac{M_z}{r}} = \sqrt{6{,}67 \cdot 10^{-11} \frac{5{,}9 \cdot 10^{24}}{(6{,}378 \cdot 10^6 + 6{,}3 \cdot 10^5)}}$$

$$v \doteq 7494 \ m \cdot s^{-1}$$

$$\omega = \frac{v}{r} = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi r}{v}$$

$$t = \frac{2 \cdot \pi \cdot (6{,}378 \cdot 10^6 + 6{,}3 \cdot 10^5)}{7494} \doteq 5876 \ s \doteq 97{,}9 \ min$$

179 PODLE 178

$$v = \sqrt{\gamma \frac{M_z}{R_z}} \qquad \frac{v}{2} = \sqrt{\gamma \frac{M_z}{(R_z + h)}}$$

$$R_z + h = \gamma \frac{M_z \cdot 4}{v^2} = \gamma \frac{M_z \, 4}{\gamma \frac{M_z}{R_z}} = 4 R_z$$

$$R_z + h = 4 R_z \Rightarrow h = 4R_z - R_z = 3 \cdot 6378$$

$$h = 19\,134 \ km$$

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$$v = \sqrt{\gamma \frac{M_z}{r}} \qquad v_z = \sqrt{\gamma \frac{M_z}{R_z}}$$

$$v_M = \sqrt{\gamma \cdot \frac{0{,}107 M_z}{0{,}53 R_z}}$$

$$\frac{v_z}{v_M} = \frac{\sqrt{\gamma \frac{M_z}{R_z}}}{\sqrt{\gamma \frac{0{,}107 M_z}{0{,}53 R_z}}}$$

$$\frac{v_z}{v_M} = \sqrt{\frac{\gamma M_z 0{,}53 R_z}{\gamma 0{,}107 M_z R_z}} = \sqrt{\frac{0{,}53}{0{,}107}}$$

$$v_M = \frac{v_z}{\sqrt{\frac{0{,}53}{0{,}107}}} = \frac{7{,}9}{\sqrt{\frac{0{,}53}{0{,}107}}} = 3{,}55 \text{ km/s}$$

183

$$v = \sqrt{\gamma \frac{M}{r}} \Rightarrow M = \frac{v^2 r}{\gamma}$$

$$M = \frac{(1{,}35 \cdot 10^3)^2 \cdot 23{,}5 \cdot 10^6}{6{,}67 \cdot 10^{-11}} = 6{,}4 \cdot 10^{23} \text{ kg}$$

185

$T_V, a_V \quad T_z, a_z$

$T_z = 1 \text{ rok}$

$a_z = 1 \text{ AU}$

$$\frac{T_1^2}{T_2^2} = \frac{a_1^3}{a_2^3}$$

$$\frac{T_z^2}{T_V^2} = \frac{a_z^3}{a_V^3} \Rightarrow$$

$$a_V = \sqrt[3]{\frac{a_z^3 \cdot T_V^2}{T_z^2}} = \sqrt[3]{\frac{1^3 \cdot 0{,}615^2}{1^2}} = 0{,}72 \text{ AU}$$

186

$$\frac{T_1^2}{T_2^2} = \frac{a_1^3}{a_2^3}$$

T_U, a_U $\quad T_Z = 1$ rok

$a_Z = 1$ AU

$$\frac{T_Z^2}{T_U^2} = \frac{a_Z^3}{a_U^3} \Rightarrow T_U = \sqrt{\frac{T_Z^2 a_U^3}{a_Z^3}}$$

$$T_U = \sqrt{\frac{1^2 \cdot 19{,}2^3}{1^3}} \doteq \underline{84{,}1 \text{ roku}}$$

187

$a_P = 0{,}308$ AU

$a_A = 0{,}466$ AU

$$\frac{T_1^2}{T_2^2} = \frac{a_1^3}{a_2^3}$$

$$\frac{T_Z^2}{T_H^2} = \frac{a_Z^3}{a_H^3}$$

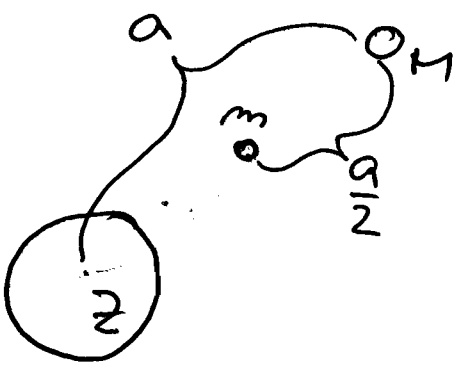
$$T_H = \sqrt{\frac{T_Z^2 a_H^3}{a_Z^3}}$$

$$a_H = \frac{0{,}308 + 0{,}466}{2} = 0{,}387$$

$$T_H = \sqrt{\frac{1^3 \cdot 0{,}387^3}{1^3}}$$

$$T_H = \underline{0{,}24 \text{ roku}}$$

(190) VYCHÁZÍME Z ÚLOHY 189



$$\frac{T_1^2}{T_2^2} = \frac{a_1^3}{a_2^3}$$

$a_1 = 1$

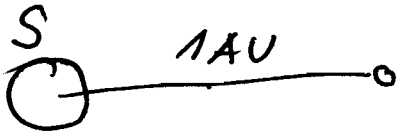
$a_2 = \frac{1}{2} a_1$

$$T_2 = \sqrt{\frac{T_1^2 \cdot a_2^3}{a_1^3}}$$

$$T_2 = \sqrt{\frac{27{,}3^2 \cdot 0{,}5^3}{1^3}} = \underline{9{,}65 \text{ dne}}$$

T_2 JE OBĚŽNÁ DOBA CELÉHO PŘÍLETU. PROTOŽE TĚLESO NARAZÍ DO ZEMĚ V $\frac{1}{2}$ ČASU JE ČAS DOPADU ROVEN $\frac{T_2}{2} \doteq \underline{4{,}83 \text{ dne}}$

(191) ŘEŠÍME JAKO PŘEDCHOZÍ ÚLOHU



$$\frac{T_1^2}{T_2^2} = \frac{a_1^3}{a_2^3}$$

$a_1 = 1$

$a_2 = \frac{1}{2} a_1$

$$T_2 = \sqrt{\frac{T_1^2 \cdot a_2^3}{a_1^3}} = \sqrt{\frac{1^2 \cdot 0{,}5^3}{1^3}} = \underline{0{,}353 \text{ roku}}$$

TĚLESO NARAZÍ JE NA POLOVINU DOBY

$$\frac{T_2}{2} = \frac{0{,}353}{2} = 0{,}\underline{1765 \text{ roku}} \doteq \underline{64{,}4 \text{ dne}}$$